Brane World Phenomenology and the Z_2 Symmetry

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Abstract

The Friedmann equation for a positive tension brane world with and without the Z_2 symmetry is derived and the possible effects of dropping the Z_2 symmetry on the expansion of our Universe are discussed. The global solutions for the metric in the infinite extra dimension case are found; cosmological constraints are discussed. This is applied to various phenomenological aspects of brane worlds, including phase transitions, topological defects and GUT baryogenesis. Important differences to the usual case are elucidated.

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I. INTRODUCTION

Recently there has been considerable interest in the novel suggestion that the physical universe is embedded in higher dimensions with standard model particles confined to a 3-brane and gravity propagating in the extra dimensions [1–4]. Randall and Sundrum [5] have even suggested that the extra dimension could be non-compact. The cosmology of these extra dimension scenarios has been investigated and the Friedmann equation derived and shown to contain important deviations from the usual 4-dimensional case [6].

Most brane world scenarios use a Z_2 symmetry about our brane. This is motivated by the work of Horava and Witten [7]. However, recent work in this area concentrates on one infinite extra dimension and is not directly derived from M-theory. If one drops the M-theory motivation there is no reason why we should necessarily assume the Z_2 symmetry about our brane. There have been phenomenological multi-brane scenarios which involve some branes that do not possess a Z_2 symmetry [8] and it is therefore interesting to entertain the possibility that we live on such a brane. In section 2 of this paper we investigate the Friedmann equation and associated cosmological consequences of such models. We show that the Friedmann equation acquires an extra term when there is no Z_2 symmetry, which can give rise to a period of expansion on the brane, and consider constraints on such a term from nucleosynthesis. We also derive the global solutions, showing they remain well-defined.

If the brane world scenario is correct then it needs to describe the world we live in. In particular, it needs to reproduce standard model physics at low energies and also lead to cosmological structure formation. Fundamental to both are phase transitions in the early universe. Since the underlying cosmology is changed in the brane picture, cosmological phase transitions could also change. In particular, in section 3 we investigate first order phase transitions in brane cosmologies. The increased expansion at early times can have dramatic consequences, which we elucidate.

Moving to a brane world picture introduces many novel features to defect cosmology. There are two broad classes of effect that need to be considered. On the microscopic scale there are changes to the physics which determines the properties of individual defects. While on the macroscopic scale, modifications to the Friedmann equation change the evolution of defect networks. A full treatment of brane world defects would require a consistent solution representing a defect on the brane. Given that heavy defects may produce strong gravitational backreactions on the brane, in general such a calculation lies outside the scope of the low-energy effective field theory formalism [3]. Some possible changes in defect microstructure due to the modifications of the gravitational interaction on small scales are considered in [9]. In this paper we consider the evolution of *standard* defects with the brane world Friedmann equation in section 4.

The increased expansion rate in the brane world scenario will lead to different freeze-out temperatures for particle interactions. This will change the abundance of particles produced at early times. An example of this is GUT baryogenesis, which we examine in section 5. Depending on the fundamental parameters, the increased expansion rate can lead to a suppression of the resulting baryon asymmetry. Our conclusions are summarised in section 6.

II. MATTER ON THE BRANE AND NO Z_2 SYMMETRY

The recent interest in brane worlds and extra dimensions has been inspired mostly by [7], in which one of the extra dimensions is larger than the others and there is a Z_2 symmetry that exists across each brane. There have, however, been attempts to construct phenomenological multi-brane models where the Z_2 symmetry has been relaxed across some branes [8]. It is thus interesting to consider the case where this symmetry condition is not assumed. Brane worlds where the Z_2 symmetry is not present have been considered in [10,11] although there they examine a brane that is sandwiched between two different spacetimes with different cosmological constants. Here instead, we relax the Z_2 condition on the solution for the metric itself. This generates an altered Friedmann equation as well as giving different bulk solutions.

A. The General Friedmann Equation

In this section we examine a cosmologically realistic positive tension 3-brane in 5 dimensions and the corresponding solution for the metric in the bulk. We derive the general solution without assuming the Z_2 symmetry about the brane. Following the setup and notation of [12] the metric takes the form,

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \tag{1}$$

and we assume immediately that $b^2(t,y)$ is not a function of time and therefore y can be scaled so that $b(y) = 1$. The metric is found by solving the 5D Einstein's equations, and we define $\kappa^2 = 1/\widetilde{M_5}^3$ where $\widetilde{M_5}$ is the fundamental (reduced) 5D Planck Mass:

$$G_{AB} = \kappa^2 T_{AB}. \tag{2}$$

The stress-energy-momentum tensor can be written as,

$$T^A{}_B = T^A{}_B|_{brane} + T^A{}_B|_{bulk}. \tag{3}$$

With a homogeneous and isotropic geometry in the brane the first term can be written as,

$$T^A{}_B|_{brane} = \frac{\delta(y)}{b}\mathrm{diag}(-\rho_b, p_b, p_b, p_b, 0), \tag{4}$$

and the second term, which describes a bulk cosmological constant, is of the form,

$$T^A{}_B|_{bulk} = \mathrm{diag}(-\rho_B, -\rho_B, -\rho_B, -\rho_B, -\rho_B). \tag{5}$$

As is shown in [12], any set of functions $a(t,y)$ and $n(t,y)$ satisfying,

$$\left(\frac{\dot{a}}{na}\right)^2 = \frac{1}{6}\kappa^2\rho_B + \left(\frac{a'}{a}\right)^2 - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}, \tag{6}$$

together with $G_{05} = 0$, will be solutions to all of Einstein's equations locally in the bulk. $G_{05} = 0$ is satisfied if

$$n(t,y) = \frac{\dot{a}(t,y)}{\dot{a}(t,0)}. \tag{7}$$

Here, we have normalised $n(t,y)$ so that $n(t,0) = 1$. In order to obtain the Friedmann Equation on the brane, we evaluate (6) at $y = 0$. This is easily done except for the $(a'/a)^2$ term. To evaluate this we need the junction conditions,

$$\frac{[a']}{a_0} = -\frac{\kappa^2}{3}\rho_b, \tag{8}$$

$$\frac{[n']}{n_0} = \frac{\kappa^2}{3}(3p_b + 2\rho_b), \tag{9}$$

where $[Q] = Q(0^+) - Q(0^-)$ and $Q(0) = Q_0$. Now instead of assuming the Z_2 symmetry $y \leftrightarrow -y$ which would give $a'(0^+) = -a'(0^-)$ and therefore $[a'] = 2a'(0^+)$, we write,

$$a'(0^+) = -a'(0^-) + d(t). \tag{10}$$

Here $d(t)$ is some function of time only and has yet to be determined. Now using (8) gives,

$$a'(0^+) = -\frac{\kappa^2}{6}\rho_b a_0 + \frac{d(t)}{2}, \tag{11}$$

$$a'(0^-) = \frac{\kappa^2}{6}\rho_b a_0 + \frac{d(t)}{2}. \tag{12}$$

Assuming that $a'^2(0) = (a'^2(0^+) + a'^2(0^-))/2$ results in,

$$\frac{a_0'^2}{a_0^2} = \frac{\kappa^4}{36}\rho_b^2 + \frac{d^2(t)}{4a_0^2}. \tag{13}$$

To find an expression for $d(t)$ we take the jump of the (5,5) component of Einstein's equations as is done in [6],

$$\frac{\bar{a}'}{a_0}p_b = \frac{1}{3}\rho_b\frac{\bar{n}'}{n_0}, \tag{14}$$

where $\bar{Q} = (Q(0^+) + Q(0^-))/2$. Replacing a and n using equations (10) and (7), shows that,

$$\dot{d(t)} = \frac{3p_b\dot{a}_0}{\rho_b a_0}\, d(t). \tag{15}$$

The energy conservation equation is derived directly from the junction conditions (8) and (9) (as shown in many of the refs),

$$\dot{\rho_b} = -3(\rho_b + p_b)\frac{\dot{a}_0}{a_0}. \tag{16}$$

By using this to solve the differential equation (15), it gives us the desired expression for $d(t)$,

$$d(t) \ = \ \frac{2F}{\rho_b a_0^3}, \tag{17}$$

where F is an integration constant. Combining this with equation (6) and doing the usual replacements to obtain standard cosmology at late times: $\rho_b = \rho + \rho_\lambda$ and $\kappa^2 \rho_B/6 + \kappa^4 \rho_\lambda^2/36 = 0$, results in the Friedmann equation for a brane without the Z_2 symmetry,

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{\kappa^4 \rho_\lambda}{18}\rho \ + \ \frac{\kappa^4}{36}\rho^2 \ - \ \frac{k}{a_0^2} \ + \ \frac{\mathcal{C}}{a_0^4} \ + \ \frac{F^2}{(\rho+\rho_\lambda)^2 a_0^8}. \tag{18}$$

So the absence of the Z_2 symmetry gives rise to an extra term in the Friedmann equation (first found by [13]). For a radiation dominated Universe where $\rho = \gamma/a_0^4$, the extra term behaves as F^2/γ^2 as $\rho \to \infty$ and $(F\rho/\gamma\rho_\lambda)^2$ as $\rho \to 0$.

In order to obtain standard cosmology at late times we need to make the identification,

$$\frac{\kappa^4 \rho_\lambda}{18} = \frac{8\pi G_4}{3} = \frac{1}{3\widetilde{M}_4^2}, \tag{19}$$

where we have used the reduced 4D Planck mass defined by $\widetilde{M}_4^2 = M_4^2/8\pi$ and will use the 5D reduced Planck mass defined by $\widetilde{M}_5^3 = M_5^3/8\pi$. This implies that the brane tension, $\rho_\lambda = 6\widetilde{M}_5^6/\widetilde{M}_4^2$. Using this and the fact that $\kappa^2 = 1/\widetilde{M}_5^3$ and also defining the dimensionless constant $f = F\widetilde{M}_4^2/\gamma\widetilde{M}_5^3$ allows us to write (18) in terms of f, $\widetilde{M}_4$ and $\widetilde{M}_5$,

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{1}{3\widetilde{M}_4^2}\rho + \frac{1}{(6\widetilde{M}_5^3)^2}\left[\rho^2 + \frac{f^2\rho_\lambda^2\rho^2}{(\rho+\rho_\lambda)^2}\right]. \tag{20}$$

This shows that the expansion of the universe is initially dominated by the ρ^2 term, while at late times the standard cosmology phase with the usual $H^2 \propto \rho$ behaviour is obtained. If $f < \sqrt{8}$ the third, 'f'-term of (20) is always less significant than the other terms, and the resulting cosmology is similar to a brane cosmology with a Z_2 symmetry.

If $f > 3$ there will be a period between the ρ^2 and ρ driven phases when the 'f'-term is dominant. To get an explicit solution for $a_0(t)$ we will make several approximations. We will ignore all subdominant terms in each phase of the universe, use,

$$\frac{\rho^2\rho_\lambda^2}{(\rho+\rho_\lambda)^2} = \begin{cases} \rho^2 & \rho < \frac{1}{4}\rho_\lambda \\ \frac{1}{4}\rho_\lambda\rho & \frac{1}{4}\rho_\lambda < \rho < 4\rho_\lambda \\ \rho_\lambda^2 & \rho > 4\rho_\lambda \end{cases}, \tag{21}$$

to approximate the 'f'-term, and take $f \gg 1$. It is useful to introduce the time scale $t_{\mathrm{F}} = \widetilde{M}_4^2/(4\widetilde{M}_5^3 f)$. The resulting evolution of the universe then divides into 5 phases as follows:

- PHASE 1 ($0 < t < t_{\mathrm{F}}$): Initially (20) is dominated by the ρ^2 term. This continues until $t = t_{\mathrm{F}}$ when $\rho = f\rho_\lambda$, after which the 'f'-term becomes dominant. Until that happens,

$$a_0(t) = \left[\frac{2\gamma}{3\widetilde{M}_5^3}\right]^{1/4} t^{1/4} \tag{22}$$

- PHASE 2a ($t_{\rm F} < t < t_{\rm F} + t_{\rm I}$), with $t_{\rm I} = t_{\rm F} \ln(f/4)$: For large ρ the 'f' term can be approximated by a constant. During this period the universe expands exponentially, as in inflation,

$$a_0(t) = \left[\frac{\widetilde{M}_4^2 \gamma}{6\widetilde{M}_5^6 fe}\right]^{1/4} e^{Ht} \ , \quad H = \frac{\widetilde{M}_5^3}{\widetilde{M}_4^2} f. \tag{23}$$

- PHASE 2b ($t_{\rm I} + t_{\rm F} < t < t_{\rm I} + 7t_{\rm F}$): For $\rho \sim \rho_\lambda$ the 'f'-term starts to decrease, and is approximately proportional to $f^2 \rho_\lambda \rho$. During this phase,

$$a_0(t) = \left[\frac{\gamma f^2}{6\widetilde{M}_4^2}\right]^{1/4} (t - t_{\rm I} + t_{\rm F})^{1/2}. \tag{24}$$

- PHASE 2c ($t_{\rm I} + 7t_{\rm F} < t < t_{\rm B} = t_{\rm I} + (3 + f^2/2)t_{\rm F}$): For small ρ the 'f'-term is approximately $f^2\rho^2$. It ceases to be the dominant term in (20) when $\rho = 2\rho_\lambda/f^2$. Until then,

$$a_0(t) = \left[\frac{2\gamma f}{3\widetilde{M}_5^3}\right]^{1/4} (t - t_{\rm I} - 3t_{\rm F})^{1/4}. \tag{25}$$

- PHASE 3 ($t > t_{\rm B}$): Finally, at late times the ρ term dominates (20) to give the standard cosmology,

$$a_0(t) = \left[\frac{4\gamma}{3\widetilde{M}_4^2}\right]^{1/4} (t - t_{\rm B} + f^2 t_{\rm F})^{1/2}. \tag{26}$$

The above solution for $a_0(t)$ presents a very different picture of the evolution of our universe: it has the unconventional early ρ^2 behaviour as seen in most brane world models, but now this is broken up by a period of exponential expansion. During this extra phase the scale factor increases by a factor of $f^{1/4}$. Like inflation this could help to solve the flatness problem, however unlike inflation it is not followed by reheating and so cannot help with the horizon and monopole problems. Eventually, as expected, the standard cosmology is obtained. The above approximate solution suggests that there will be no exponential expansion unless $f \gtrsim 4$. If f is very large ($f \gtrsim \widetilde{M}_4^2/\widetilde{M}_5^2$) then $f\rho_\lambda > \widetilde{M}_5^4$ and Phase 1, and some of the succeeding phases will be above the 5D Planck scale, and may not actually occur.

For $\sqrt{8} < f < 3$ there will be a short period of 'f'-term domination. This occurs after the time when the ρ-term of (20) starts to dominate the ρ^2-term. At this stage the 'f'-term is no longer approximately constant, and so there is no exponential expansion.

We can obtain rough restrictions on f in terms of $\widetilde{M}_5$ by demanding that standard cosmology is in place by the time of nucleosynthesis. This is equivalent to requiring that phase 2 is over well before nucleosynthesis begins, which implies that,

$$\frac{1}{3\widetilde{M}_4^2}\rho_N \gg \frac{1}{(6\widetilde{M}_5^3)^2} f^2 \rho_N^2, \tag{27}$$

where ρ_N is the energy density at t_N, the time of nucleosynthesis. At this time the universe is radiation dominated and ρ_N can be written in terms of the temperature at nucleosynthesis, $T_N \approx 1$MeV, and the number of effective relativistic degrees of freedom, g_*, which, in the standard model is 10.75;

$$\rho_N = g_* \frac{\pi^2}{30} T_N^4. \tag{28}$$

Substituting this into (27) and switching to the standard 4D and 5D Planck masses, gives the following relation between the 5D Planck mass M_5 and the Z_2 symmetry breaking parameter f,

$$M_5 \gg \left(\frac{g_* \pi^3}{45} T_N^4 M_4^2 f^2\right)^{1/6} \approx 30 f^{1/3}\text{TeV}. \tag{29}$$

The corresponding result for $f = 0$ is,

$$M_5 \gg 30\text{TeV}. \tag{30}$$

This is a fairly weak bound on M_5. For the infinite extra dimension scenario, experiments testing gravity at small distances have already demonstrated using the corrections to Newton's gravity law calculated by Randall and Sundrum [5] that $M_5 > 10^5$TeV. This experimental constraint however, is not applicable to compactified scenarios. Supposing that we do live in an infinite 5th dimension and that M_5 has a value that is just outside our experimental reach, we can then use (29) to constrain f; $f \ll 10^{11}$. In this case, the period of exponential expansion (Phase 2a) would only last for $t_{\rm I} = M_4^2/(4M_5^3 f)\ln f \simeq 10^7\text{TeV}^{-1}$ and the scale factor would increase during this time by a factor of around 500. Increasing M_5 relaxes the bound on f and would appear to lead to more inflation. However, if M_5 is too large then some, or all, of the resulting expansion occurs at energies higher than the 5D Planck scale. Consequently the maximum inflation occurs for $M_5 = 5 \times 10^6$ TeV and $f = 10^{17}$, which leads to an expansion of only 10^4. Unfortunately, this is not cosmologically relevant. Note that, from equations (11), (12), (17) and (29) the effect of the Z_2 breaking term decreases with increasing time such that the universe reverts to standard cosmology. This suggests that brane world scenarios where the physical Universe is on a brane without this symmetry, for example [14], are not viable after nucleosynthesis.

B. Global Solutions

Before examining the phenomenology of brane worlds further, we will first solve the 5D Einstein equations in the bulk and hence derive the corresponding global solutions for $a(t, y)$ and $n(t, y)$ for the non-Z_2 symmetric situation in an infinite extra dimension. We derive this solution for the general case first and then for a specific cosmologically realistic brane. To do this we can adapt the previously known general global solution for a brane with tension ρ_b and negative 5D bulk cosmological constant ρ_B to the non-Z_2 symmetric case.

We know from the (0,0) component of Einstein's equations that the new non-symmetric solution will have a form similar to the symmetric case [12];

$$a^2(t,y) = a_0^2\left(A(t)\cosh\mu y + B(t)\sinh\mu|y| + C(t)\right), \tag{31}$$

where $\mu = \sqrt{\frac{-2\rho_B}{3\widetilde{M}_5^3}}$. The requirement $a^2(t,0) = a_0^2$ trivially implies that $A(t) + C(t) = 1$ for all t. Now using equation (11) it can be seen that,

$$a'(t,0^\pm) = \left\{a'_{sym}(t,0^\pm)\right\} + \frac{d(t)}{2}, \tag{32}$$

$$\Rightarrow \quad B(t) = \left\{-\frac{\rho_b}{\sqrt{-6\widetilde{M}_5^3\rho_B}}\right\} \pm \frac{d(t)}{a_0\mu}. \tag{33}$$

Here the $\pm$ in the expression for $B(t)$ corresponds to the solution on either side of the brane and we use $\{\ldots\}$ to denote the solution found in the Z_2 symmetric case.

$C(t)$ is found from the differential equation for $a^2(t,y)$ which is derived from the Einstein equations, (see [12]),

$$C(t) = \frac{3\widetilde{M}_5^3(\dot{a}_0^2 + k)}{\rho_B a_0^2}. \tag{34}$$

Rewriting this using the 'new' Friedmann equation (18) gives,

$$C(t) = \left\{\frac{1}{2}\left(1 + \frac{\rho_b^2}{6\widetilde{M}_5^3\rho_B}\right) + \frac{3\widetilde{M}_5^3\mathcal{C}}{\rho_B a_0^4}\right\} + \frac{3\widetilde{M}_5^3 F^2}{\rho_B\rho_b^2 a_0^8}, \tag{35}$$

and therefore $A(t)$ is trivially given by,

$$A(t) = \left\{\frac{1}{2}\left(1 - \frac{\rho_b^2}{6\widetilde{M}_5^3\rho_B}\right) - \frac{3\widetilde{M}_5^3\mathcal{C}}{\rho_B a_0^4}\right\} - \frac{3\widetilde{M}_5^3 F^2}{\rho_B\rho_b^2 a_0^8}. \tag{36}$$

Using (32) and (17) leads to,

$$B(t) = \left\{-\frac{\rho_b}{\sqrt{-6\widetilde{M}_5^3\rho_B}}\right\} \pm \sqrt{\frac{6\widetilde{M}_5^3}{-\rho_B}}\frac{F}{\rho_b a_0^4}. \tag{37}$$

Again the $\pm$ signs in the expression for $B(t)$ give the two different solutions on either side of the brane. The solution for $n(t,y)$ in the non-symmetric case is found from the above solution for $a(t,y)$ by using equation (7) as before. It is easily seen that setting F to zero in the above solutions recovers the Z_2 symmetric situation.

We are interested in these solutions for a cosmologically realistic brane, so we make the same substitutions as were made to generate (18) and also assume a radiation dominated Universe. Setting $\rho = \gamma/a_0^4$ where γ is a constant, we obtain expressions for $A(t)$, $B(t)$ and $C(t)$ corresponding to a brane with a viable cosmology,

$$A(t) = 1 + \chi + \frac{1}{2}\chi^2 + +c\chi + \frac{f^2\chi^2}{2\left(1+\chi\right)^2}, \tag{38}$$

$$B(t) = -(1+\chi) \pm \frac{f\chi}{(1+\chi)}, \tag{39}$$

$$C(t) = -\chi - \frac{1}{2}\chi^2 - c\chi - \frac{f^2\chi^2}{2\left(1+\chi\right)^2}. \tag{40}$$

Where we have defined $\chi = \widetilde{M}_4^2\rho/6\widetilde{M}_5^6$, and $c = 3\widetilde{M}_4^2\mathcal{C}/\gamma$

While it is possible for $a(t,y)$ or $n(t,y)$ to vanish, calculation of the Ricci tensor and scalar show that there is merely a coordinate singularity at these points. A similar result has been obtained in [15] for the symmetric case. Thus the only clear restriction on f is that of (29) which is applicable to any brane world scenario that has broken Z_2 symmetry across our brane, except the class of models that involve bulk matter.

III. PHASE TRANSITIONS IN BRANE COSMOLOGY

Having derived the general Friedmann equation without assuming a Z_2 symmetry about our brane, we now turn to its applications in physical situations. If the brane world scenario is to describe the Universe we live in then it must reproduce standard model physics. It must also give rise to a mechanism for structure formation. In this section we consider phase transitions in a brane world; in particular first order phase transitions. In a cosmological setting, these could be modified because of the revised Friedmann equation. We also consider the possibility that M_5 is just above the GUT scale, and consider cosmological GUT phase transitions. These can give rise to topological defects. We examine whether or not the properties of such resulting defects are modified in a brane world scenario.

Assuming the F, $\mathcal{C}$ and k terms in (18) are all negligible, the solution of the brane cosmology Friedmann equation leads to the non-standard temperature-time relation,

$$t = \frac{45}{8\pi^3}\frac{M_5^3}{g_* T^4}, \tag{41}$$

for $T > T_{\rm B}$ and

$$t = \frac{3\sqrt{5}}{4\pi^{3/2}}\frac{M_4}{g_*^{1/2}T^2} - t_{\rm B}, \tag{42}$$

for $T < T_{\rm B}$, where $T_{\rm B} = [45M_5^6/(g_*\pi^3 M_4^2)]^{1/4}$ and $t_{\rm B} = M_4^2/(8M_5^3)$ are the temperature and time at the end of the brane era. This is in contrast with the usual relation (which is simply (42) with $t_{\rm B} = 0$). At early times the universe cools far more rapidly in the brane cosmology as the Hubble parameter at any given temperature is much greater in this model.

During a phase transition, bubbles of true vacuum will nucleate and expand. Assuming they expand at the speed of light, the fraction of space remaining in the false vacuum at time t is [16],

$$p(t) = \exp\left\{-\frac{4\pi}{3}\int_0^t dt_1 a_0^3(t_1)\Gamma(t_1)\left[\int_{t_1}^t \frac{dt_2}{a_0(t_2)}\right]^3\right\}. \tag{43}$$

The three factors in the integral are respectively the red shift, the bubble nucleation rate, and the volume at time t of a bubble which formed at time t_1. The probability per unit time and volume that a critical size bubble will nucleate can be approximated by $\Gamma(T) = \nu T^4 \theta(T_c - T)$. The parameter ν will depend on the expansion rate [17] as well as the details of the scalar potential, thus it will be different in the two models.

The number of bubbles nucleated by time t is given by,

$$n_{\text{bubble}} = a_0^{-3}(t) \int_0^t dt_1 a_0^3(t_1) \Gamma(t_1) p(t_1). \tag{44}$$

A. Standard Cosmology

Evaluating the integrals in (43) in the standard cosmology gives,

$$p(T) = \exp\left\{ -\frac{25\nu}{3\pi g_*^2} \left(\frac{3M_4}{2\pi}\right)^4 \left(\frac{1}{T} - \frac{1}{T_c}\right)^4 \right\} . \tag{45}$$

If the end of the phase transition is taken to be when $p = 1/2$, this will occur at $T = T_*$ given by,

$$\frac{1}{T_*} = \frac{1}{T_c} + \frac{1.5 g_*^{1/2}}{M_4 \nu^{1/4}} . \tag{46}$$

For a GUT transition $T_c \ll M_4$ and so unless ν is very small $T_* \approx T_c$.

Evaluating (44) reveals that the number of bubbles nucleated rapidly tends to $0.9\nu^{3/4}T^3$ after the end of the transition.

B. Brane Cosmology

Using (41) to evaluate (43), assuming $T_c > T_B$, gives,

$$p(T) = \exp\left\{ -\frac{\pi\nu}{3} \left(\frac{15M_5^3}{2\pi^3 g_*}\right)^4 \left(\frac{1}{T^3} - \frac{1}{T_c^3}\right)^4 \right\}, \tag{47}$$

and so this time the transition ends when,

$$\frac{1}{T_*^3} = \frac{1}{T_c^3} + \frac{3.7 g_*}{M_5^3 \nu^{1/4}}. \tag{48}$$

If T_c is close to M_5, then even for moderate values of ν the second term of (48) can be the dominant one. In this case the transition will be significantly slower and involve a greater temperature drop. The number of bubbles nucleated (44) has the same large t behaviour as in the standard cosmology.

C. Avoiding Vacuum Domination

If the phase transition goes too slowly, it is possible that false vacuum energy will dominate the radiation before the transition finishes. The Universe will then start to inflate, and never stop.

We will consider the simple Higgs model with an effective potential

$$V(\phi, T) = \frac{\lambda}{4}\left[\phi^2 + \frac{T^2}{4} - \frac{T_c^2}{4}\right]^2 \tag{49}$$

The false vacuum energy density (when $\phi = 0$) is then

$$\rho_{\rm FV} = \frac{\lambda}{64}(T_c^2 - T^2)^2 p(T), \tag{50}$$

To avoid false vacuum domination need $\rho_{\rm FV} < \rho_{\rm radiation}$ hence,

$$\frac{15\lambda}{32\pi^2 g_*}p(T) < \left(\frac{T}{T_c}\right)^4 . \tag{51}$$

Vacuum domination will only occur if $\nu \ll (T_c/M_4)^4$, i.e. if the second term of (46) is the most significant. Using this assumption, the bound on ν for the standard cosmology is

$$\nu \geq 0.1 g_* \lambda \left(\frac{T_c}{M_4}\right)^4 . \tag{52}$$

In brane cosmology, assuming $\nu \ll (T_c/M_5)^{12}$, the corresponding bound is

$$\begin{aligned} \nu &\geq 4 \times 10^{-3} g_* \lambda^3 \left(\frac{T_c}{M_5}\right)^{12} \\ &\approx 8 \times 10^{-3} g_* \lambda \left(\frac{T_c}{M_4}\right)^4 \left(\frac{\lambda T_c^4}{g_* T_B^4}\right)^2 . \end{aligned} \tag{53}$$

Thus unless $T_c < (g_*/\lambda)^{1/4} T_B$ phase transitions in the brane cosmology require a higher nucleation rate to complete successfully. Of course if $T_c < T_B$ the phase transition will happen when brane effects are not significant, and the bound is given by (52). In addition, the faster expansion of the universe during the brane era will allow smaller bubbles to survive, thus the bubble nucleation rate will be increased and the bounds on the underlying parameters of the theory will be weaker.

IV. DEFECTS IN BRANE COSMOLOGY

A natural result of cosmological phase transitions are topological defects. If, after a phase transition, the vacuum manifold has non-trivial homotopy groups, topological defects will form in brane cosmology, just as they do in standard cosmology. As in the normal case, defects can have potentially useful (and sometimes disastrous) cosmological implications. For example, GUT scale cosmic strings can lead to a realistic scenario for structure formation. However, magnetic monopoles and domain walls rapidly dominate the energy density of the universe. We examine whether or not the properties of such resulting defects are modified in a brane world scenario. To evaluate their properties we need to find the initial defect density and then determine how the density evolves.

For a first order phase transition the initial correlation length of the defects is easily determined from the arguments in the previous section,

$$\xi \sim \frac{1}{\lambda n_{\rm bubble}^{1/3}} \sim \frac{1}{\lambda T_*} \ . \tag{54}$$

Next we consider the evolution of defects in brane cosmology.

A. Shadowing verses Scaling

An immediate consequence of the modified brany evolution is the different relationship between scaling (i.e. a fixed number of defects per horizon volume) and shadowing (i.e. defect density remaining a fixed fraction of the dominant energy density).

For scaling defects in either model we have,

$$\rho_{\rm string} \propto \frac{ct}{(ct)^3} \propto t^{-2} \ , \quad \rho_{\rm wall} \propto \frac{(ct)^2}{(ct)^3} \propto t^{-1} \ . \tag{55}$$

If the dominant energy density varies as a^{-w}, we saw above that $a \propto t^{1/w}$ in the brane era and $a \propto t^{2/w}$ in the normal picture. Thus in the brane era we have,

$$\rho_{\rm dominant} \propto t^{-1} \ , \tag{56}$$

while in the normal picture we have,

$$\rho_{\rm dominant} \propto t^{-2} \ . \tag{57}$$

In the standard picture, scaling strings shadow the dominant energy density, while in the brane era scaling walls shadow the dominant energy density.

B. Monopoles

At formation $n_{\rm monopole} \sim n_{\rm bubble}$. Red-shifting gives $n_{\rm monopole} \sim T^3$ at later times, thus in the absence of annihilation, both cosmologies would predict the same monopole number in the current universe. While monopole annihilation could look very different in brane cosmology if the brane era were persistent, the limited duration of the brane epoch curtails annihilation.

As for any 2-body annihilation process, the number density of monopoles relative to photons, $r_M = n_M/n_\gamma$ is governed by,

$$\frac{dr_M}{dt} = -\beta_M n_\gamma (r_M^2 - r_{M_{eq}}^2) \ , \tag{58}$$

where β_M parameterises the monopole annihilation rate and $r_{M_{eq}}$ is the equilibrium monopole to photon ratio.

Let us assume that $r_{M_{eq}}$ rapidly drops to zero and set $n_\gamma = \alpha T^3$. We can take the temperature-time relationship to be $T = Ct^{-1/w}$, where C, α and w are constants. If the monopoles form at time t_f at a density r_{M_f}, integration gives,

$$r_M^{-1} = \beta_M \alpha C^3 \frac{t^{1-3/w} - t_f^{1-3/w}}{1-3/w} + r_{M_f}^{-1} \ . \tag{59}$$

In the standard picture, for a radiation dominated Universe, $w = 2$ and at large times we have the standard freeze out picture with,

$$r_M^{-1}|_{\text{f.o.}} = 2\beta_M \alpha C^3 t_f^{-1/2} + r_{M_f}^{-1} \ . \tag{60}$$

Clearly, freeze out only occurs for $w < 3$, for $w > 3$, r_M decays with time. (For $w = 3$, r_M decays like $1/\log(t)$ at late times) Thus naively things look very different in the brane era. Here $w = 4$ and

$$r_M^{-1} = 4\beta_M \alpha C^3 (t^{1/4} - t_f^{1/4}) + r_{M_f}^{-1} \ . \tag{61}$$

At large times, $r_M \propto t^{-1/4}$, instead of freeze out, the monopole density continues to decay and there would appear to be no monopole problem. However, this result does not survive the inclusion of constants and the inevitable termination of the brane era.

Let the brane era persist well beyond the GUT time, then

$$r_M^{-1} \simeq 4\beta_M \alpha C^3 t^{1/4} + r_{M_f}^{-1} \ . \tag{62}$$

If we denote the time and temperature of the brane-normal transition by t_{B} and T_{B}, we have,

$$t_f = t_{\text{B}} \left(\frac{T_{\text{B}}}{T_f}\right)^w \ . \tag{63}$$

If we now look at the monopole density at t_{B}, in the brane model we have,

$$r_M^{-1} \simeq 4\beta_M \alpha T_{\text{B}}^3 t_{\text{B}} + r_{M_f}^{-1} \ . \tag{64}$$

While in the normal model we have,

$$r_M^{-1}|_{\text{f.o.}} = 2\beta_M \alpha T_{\text{B}}^3 t_{\text{B}} \left(\frac{t_{\text{B}}}{t_f}\right)^{1/2} + r_{M_f}^{-1} \ . \tag{65}$$

Given that $t_{\text{B}} \gg t_f$, we see that the annihilation in the normal model is far more efficient than in the brane model: the transient nature of the brane era and the constants conspire to over turn the naive expectations from the proportionalites.

C. Cosmic Strings

At early times in their evolution cosmic strings experience a significant damping force from the background radiation density. For strings the damping force is the dominant effect when $T^3/T_{\rm c}^2 > H$, in which case $\xi \propto T_{\rm c} T^{-3/2} t^{1/2}$ [18]. In the standard cosmology the correlation length is,

$$\xi \sim \frac{T_{\rm c} M_4^{1/2}}{T^{5/2}} \sim \frac{T_{\rm c}}{M_4^{3/4}} t^{5/4}, \tag{66}$$

and $\rho_{\rm string} = T_{\rm c}^2/\xi^2$. This continues until $T \sim T_{\rm c}^2/M_4$, after which the evolution will start to approach a scaling solution ($\xi \sim t$).

During the initial period of non-standard evolution in the brane cosmology the string evolution is always friction dominated. This continues until $T \sim T_{\rm c}^2/M_4$, as in the standard cosmology.

For $T > T_{\rm B}$ the string density satisfies,

$$\xi \sim \frac{T_{\rm c}}{M_5^{3/2} T^{7/2}} \sim \frac{T_{\rm c}}{M_5^{9/8}} t^{7/8}. \tag{67}$$

Using the relationship between $T_{\rm B}$, M_5 and M_4, we find that the correlation length at $T = T_{\rm B}$ in the brane cosmology is of the same order as in the standard cosmology. For $T < T_{\rm B}$ the correlation length evolves as in the standard case, thus the string density at the end of friction domination is the same in both pictures.

D. Domain Walls

The early evolution of domain walls can also be friction dominated. The correlation length is $\xi \sim vt$, where v is the speed of the walls. During friction domination the wall tension and friction will of the same order. This determines the speed of the walls: $v^2 \sim T_{\rm c}^3/(tT^4)$ [18]. Now $\rho_{\rm wall} \sim T_{\rm c}^3/\xi$ so $\rho_{\rm wall}/\rho_{\rm rad.} \sim v$. This means that when the walls become relativistic they will also start to dominate the energy density of the Universe.

In the standard cosmology the velocity of the walls is,

$$v \sim \frac{\rho_{\rm wall}}{\rho_{\rm rad.}} \sim \frac{T_{\rm c}^{3/2}}{M_4^{1/2} T} \ . \tag{68}$$

Hence the walls dominate the Universe at $T = (T_{\rm c}/M_4)^{1/2} T_{\rm c}$.

During the early brane cosmology,

$$v \sim \frac{\rho_{\rm wall}}{\rho_{\rm rad.}} \sim \left(\frac{T_{\rm c}}{M_5}\right)^{3/2} = \text{constant} \ . \tag{69}$$

Thus the domain wall energy density initially scales like radiation. After $T = T_{\rm B}$ they will behave as in the standard picture.

V. GUT BARYOGENESIS

Finally we investigate the possibility of baryogenesis in a brane world. If M_5 is sufficiently large, for example just above the GUT scale, then we could consider a GUT phase transition in the brane world. In this section we investigate the resulting modifications to the usual picture which might result from the revised Friedmann equation.

In general there are three things which are required for successful baryogenesis [19]: (1) baryon number violation, (2) C and CP violation and (3) departure from thermal equilibrium. The third requirement can be illustrated with a simple generic model in which the baryon asymmetry is produced by the decay of GUT bosons (X, $\bar{X}$) [20]. At high temperatures ($T \gtrsim m_X$) the X-bosons behave relativistically and so $n_X = n_{\bar{X}} \simeq n_\gamma$ in equilibrium. If the X-bosons are still in thermal equilibrium for $T \lesssim m_X$, $n_X = n_{\bar{X}} \simeq (m_X/T)^{3/2} \exp(-m_X/T) \ll n_\gamma$, and so when they eventually decay they will produce exponentially few baryons. On the other hand if the X-bosons decouple before $T \sim m_X$ there will be n_γ of them to decay into baryons. In terms of the possible annihilation and decay processes, baryogenesis arises from the single particle decay of X and $\bar{X}$'s rather than $X\bar{X}$ annihilation.

The interactions which determine the effectiveness of the above mechanism are the decays and inverse decays of X-bosons, and $2 \leftrightarrow 2$ X-mediated B-nonconserving scatterings between baryons. For $T \lesssim m_X$ rates of these processes are respectively,

$$\Gamma_D \simeq \alpha m_X, \tag{70}$$

$$\Gamma_{ID} \simeq \alpha m_X \left(\frac{m_X}{T}\right)^{3/2} \exp\left(\frac{-m_X}{T}\right), \tag{71}$$

and,

$$\Gamma_{BNC} \simeq A\alpha^2 \frac{T^5}{m_X^4}, \tag{72}$$

where α measures the coupling strength of the X-boson and A is a large numerical factor which accounts for the number of scattering channels. If the inverse decays or the $2 \leftrightarrow 2$ baryon scatterings are still significant for $T \lesssim m_X$ the final baryon asymmetry will be suppressed.

It is convenient to define the parameters $\chi = m_X/T$ and $K = \Gamma_D/H|_{\chi=1}$. In the standard cosmology,

$$K \simeq \frac{\alpha M_4}{g_*^{1/2} m_X}\ , \tag{73}$$

while in the brane cosmology,

$$K \simeq \frac{\alpha M_5^3}{g_* m_X^3}\ . \tag{74}$$

The ratios of the interaction rates to the Hubble parameter are then,

$$\Gamma_{ID}/H \simeq K\chi^{w+3/2}e^{-\chi} \ , \tag{75}$$

$$\Gamma_{BNC}/H \simeq KA\alpha\chi^{w-5} \ , \tag{76}$$

where $w = 2$ for the standard cosmology and $w = 4$ for the brane cosmology.

If $K \ll 1$ then $\Gamma_{ID}/H < 1$ and $\Gamma_{BNC}/H < 1$ at $\chi = 1$, thus the X-bosons decouple when they are relativistic and the baryon asymmetry (B) will be maximal. If each X decay produces a mean net baryon number ϵ, then the final baryon number to entropy ratio produced when $K \ll 1$ will be,

$$B = \frac{n_b - n_{\bar{b}}}{g_* n_\gamma} \simeq \frac{\epsilon}{g_*} \ . \tag{77}$$

The C and CP violation parameter ϵ is of order α^N, where $N \geq 1$ since this is not a tree level process.

This baryon asymmetry can be damped if either the inverse decays or the baryon non-conserving scatterings persist beyond $\chi = 1$. If $1 \lesssim K \lesssim K_C$ (where K_C is a theory dependent constant), the inverse decays will still be significant for $\chi < 1$. This continues until the inverse decays freeze out with $\Gamma_{ID}/H \simeq 1$ at $\chi = \chi_f$. Approximate integration of the Boltzmann equation in this case gives,

$$B \simeq \frac{\epsilon}{g_* K \chi_f^{w-1}} \ . \tag{78}$$

For large K, χ_f has a slow, logarithmic dependence on K and the baryon asymmetry falls roughly as the inverse of K.

If $K \gtrsim K_C$ the B-nonconserving scatterings will provide the dominant damping mechanism. The freeze out for these interactions is determined by $\Gamma_{BNC}/H \simeq 1$. In this case,

$$B \simeq \frac{\epsilon}{g_*}\chi_f^2 e^{-\chi_f}. \tag{79}$$

Thus for large K the baryon asymmetry is exponentially suppressed as expected. K_C is determined by the value of K that gives simultaneous freeze out of both the inverse decays and the baryon non-conserving scatterings.

To compare the two cases we will consider typical GUT parameters: $g_* = 200$, $A = 10^3$, gauge coupling strength $\alpha_G = 1/45$ and Higgs coupling strength $\alpha_H = 10^{-3}$.

In the standard cosmology these parameters give,

$$K \sim \begin{cases} 10^{-3} M_4/m_X & \text{gauge} \\ 10^{-4} M_4/m_X & \text{Higgs} \end{cases} , \quad K_C \sim \begin{cases} 130 & \text{gauge} \\ 7000 & \text{Higgs} \end{cases} . \tag{80}$$

Assuming $m_X \gtrsim 10^{14}$GeV, there is no damping of B in the Higgs mediated case, while in the gauge mediated case B is power law damped. The CP violation required to obtain the observed baryon density, $B \sim 10^{-10}$, is given by,

$$\epsilon \sim \begin{cases} 10^{-7}(10^{16}\text{GeV}/m_X) & \text{gauge} \\ 10^{-8} & \text{Higgs} \end{cases} . \tag{81}$$

The corresponding values of K and K_C in the brane cosmology are,

$$K \sim \begin{cases} (M_5/20m_X)^3 & \text{gauge} \\ (M_5/60m_X)^3 & \text{Higgs} \end{cases}, \quad K_C \sim \begin{cases} 0.5 & \text{gauge} \\ 20 & \text{Higgs} \end{cases}. \tag{82}$$

Thus in the gauge mediated case, unless m_X is within an order of magnitude of M_5, the baryon asymmetry will be exponentially suppressed. In the Higgs case m_X must be within two orders of magnitude of M_5 to give significant baryogenesis. This change renders GUT baryogenesis particularly sensitive to an early brane era. If $m_X \sim 10^{14}$GeV, baryogenesis occurs in the brane era for M_5 as high as 10^{16}GeV.

However, this result was obtained using typical standard cosmology GUT values. If M_5 were far lower than the usual Planck scale, then the GUT scale would also have to be reduced. α would also have to be substantially smaller in order to avoid breaking the experimental bounds on the proton lifetime. This would result in a lower ϵ, further constraining the model.

VI. CONCLUSIONS

In this paper we have considered the cosmological implications of brane world models both with and without a Z_2 symmetry across the brane. We have derived the generalized Friedmann equation on the brane and studied the evolution of the scale factor. We applied this to various phenomenological processes including the progress of phase transitions, the evolution of defects and baryogenesis.

Removing the Z_2 symmetry introduces an extra term in the Friedmann equation which contributes to the effective cosmological constant at early times. This can introduce a period of exponential expansion during the early *brany* evolution. Since the size of the Z_2 breaking term is constrained by nucleosynthesis, the maximum expansion obtained in this phase is a factor of 10^4. We also note that the Z_2 breaking term decreases with time, suggesting that the universe evolves as in the symmetric case at late times. This suggests that the scenarios without this symmetry at late times are not viable. The global solution in the non-symmetric case remains well defined.

Due to the modified Friedmann equation, the rate of expansion of the Universe is increased at early times. This has important phenomenological consequences. For example, as discussed in section 3, first order phase transitions require a higher nucleation rate in order to complete, which could result in more supercooling. Indeed, if the nucleation rate is not high enough, the Universe becomes dominated by the false vacuum and the transition does not complete.

Processes that rely on interactions freezing out are also sensitive to the enhanced expansion rate. For example, this has important consequences for GUT baryogenesis. As shown in section 5, unless the mass of the relevant GUT particle is within two orders of magnitude of the fundamental Planck scale, the baryon excess is exponentially suppressed. More generally, species abundances may well be affected in the brane world picture, and require further investigation.

Defect evolution is also modified during the brane epoch. However, due to the transient nature of this phase, the current defect densities are largely unchanged. This suggests that the usual mechanism for defect inspired structure formation is largely unchanged. It also suggests that, despite the increased expansion rate at early times, the usual monopole problem associated with GUT models remains.

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